

# KIN 610: Quantitative Methods in Kinesiology

## Chapter 7: The Normal Distribution

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### 1 FYI

This presentation is based on the following books. The references are coming from these books unless otherwise specified.

#### Main sources:

- Moore, D. S., Notz, W. I., & Fligner, M. (2021). *The basic practice of statistics* (9th ed.). W.H. Freeman.
- Field, A. (2018). *Discovering statistics using IBM SPSS statistics* (5th ed.). SAGE Publications.
- Furtado, O., Jr. (2026). *Statistics for movement science: A hands-on guide with SPSS* (1st ed.). <https://drfurtado.github.io/sms>

#### ClassShare App

You may be asked in class to go to the ClassShare App to answer questions.

- <https://classsharedrfurtado.netlify.app/>

#### SPSS Tutorial

- [SPSS Tutorial: Testing Normality and Working with Distributions](#)

### 2 Intro Question

- If you imagine measuring something like height, where do you think most of the values would fall relative to the mean (average)? Near it, far from it, or evenly spread out?

Click to reveal answer

Most values would cluster around the mean, with fewer values as you move further away.

- The **normal distribution** (or bell curve) is a fundamental concept in statistics that describes how data are distributed around a central value. In this chapter, we'll explore the properties of the normal distribution, how to assess whether our data follow this pattern, and why it matters for statistical analysis in movement science.

### 3 Interactive Demonstration: Sampling from a Normal Distribution

Try generating samples from a normal distribution to see how the histogram changes with different sample sizes and parameters. This demonstrates the concept that most values cluster around the mean.

**Interactive Demo:** [Click here to open the interactive normal distribution sampling demo](#)

*Note: The interactive demo opens in a new tab/window. It starts with kinesiology-relevant defaults ( = 50 cm, = 10 cm, like jump height) but allows you to adjust all parameters to see how the histogram changes.*

- This interactive demo allows students to explore how sample size affects the appearance of the distribution.
- Key observations: With small  $n$ , histograms are irregular; with large  $n$ , they closely match the theoretical curve.
- The sample statistics show natural variation around the population parameters.

### 4 Learning Objectives

By the end of this chapter, you should be able to:

- Describe the properties of the normal distribution and its role in statistical inference
- Use z-scores to compute probabilities and percentiles for normal distributions
- Interpret skewness and kurtosis as measures of distributional shape
- Compute and interpret z-skew and z-kurtosis to assess statistical significance
- Assess normality using visual methods (histograms, Q-Q plots) and formal tests
- Recognize common patterns of non-normality in Movement Science data
- Make informed decisions about when departures from normality are consequential
- Integrate multiple lines of evidence (visual + formal) when conflicts arise

## 5 Symbols

Symbol	Name	Pronunciation	Definition
$\mu$	Population mean	“myoo”	Center of the distribution
$\sigma$	Population standard deviation	“sigma”	Spread of the distribution
$z$	Z-score	“zee”	$(x - \mu)/\sigma$
$\Phi(z)$	Cumulative probability	“phi of z”	Area under curve to the left of z
$P(X \leq x)$	Probability	“probability of X less than or equal to x”	Probability that X is less than or equal to x
Skewness	Skewness	“skew-ness”	Measure of asymmetry; 0 for symmetric
Kurtosis	Kurtosis	“kur-toh-sis”	Measure of tail weight; 0 for normal (excess kurtosis)
$z_{\text{skew}}$	Z-score for skewness	–	Skewness / SE of skewness
$z_{\text{kurt}}$	Z-score for kurtosis	–	Kurtosis / SE of kurtosis
$Q_1$	First quartile	–	25th percentile
$Q_3$	Third quartile	–	75th percentile

## 6 Introduction: The Bell Curve

The **normal distribution** (also called the bell curve or Gaussian distribution) is a continuous probability distribution that is central to statistical theory and practice<sup>[1]</sup>.

- Many inferential procedures (t-tests, ANOVA, regression) assume normality of errors or sampling distributions
- Provides a mathematical model connecting z-scores to probabilities
  - It is the basis for the Central Limit Theorem, which justifies using normal-based methods for large samples regardless of the underlying population distribution
- Serves as a reference model for interpreting performance and establishing normative ranges<sup>[2]</sup>

- Example: Vertical jump heights in a homogeneous athletic population often approximate normality, allowing coaches to identify typical vs. exceptional performance.

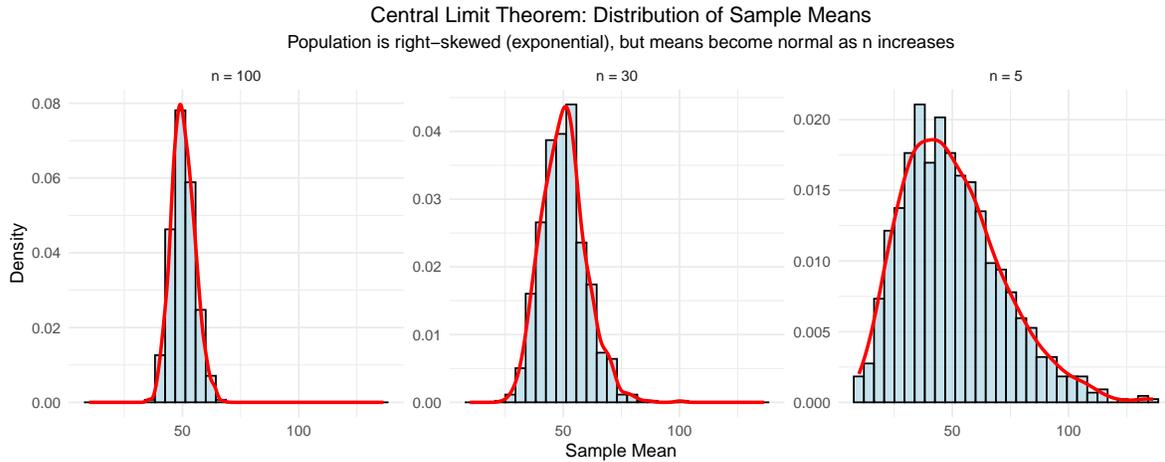


Figure 1: Central Limit Theorem demonstration: Sampling distributions of means from a right-skewed population

**! Important**

The normal distribution is a **theoretical ideal**, not a universal law of nature. Real movement data often deviate from normality in meaningful ways<sup>[3,4]</sup>.

- Opening: The normal distribution is ubiquitous in statistics but must be applied thoughtfully.
- Key question: Not “Are my data perfectly normal?” (they never are), but “Is the departure consequential for my analysis?”<sup>[5]</sup>
- Example: Sprint times in homogeneous groups approximate normality; reaction times are systematically right-skewed<sup>[6]</sup>.

## 7 Key Terms

Understanding terminology is essential for normality assessment<sup>[1,7]</sup>:

- **Normal distribution:** Symmetric, bell-shaped probability distribution defined by mean ( $\mu$ ) and SD ( $\sigma$ )
- **Standard normal distribution:** Normal distribution with  $\mu = 0$  and  $\sigma = 1$  (z-distribution)
- **68-95-99.7 rule:** ~68% within  $\pm 1\sigma$ , ~95% within  $\pm 2\sigma$ , ~99.7% within  $\pm 3\sigma$

- **Skewness:** Measure of asymmetry (0 = symmetric, positive = right tail, negative = left tail)
- **Kurtosis:** Measure of tail weight (0 = normal-like, positive = heavy tails, negative = light tails)
- **Q-Q plot:** Quantile-quantile plot comparing observed data to theoretical normal distribution
- **Shapiro-Wilk test:** Formal statistical test of normality (H : data are normal)
- **Emphasize:** These concepts work together to provide a complete picture of distributional shape.
- **Quick check:** “What does positive skewness mean?” (Right tail, mean > median)

## 8 Properties of the Normal Distribution

The normal distribution has five defining characteristics<sup>[1]</sup>:

1. **Symmetry:** Perfectly symmetric around  $\mu$ ; mean = median = mode
  2. **Unimodal:** Single peak at the mean
  3. **Asymptotic tails:** Tails extend to  $\pm\infty$ , approaching but never touching zero
  4. **Total area = 1:** Represents 100% probability
  5. **68-95-99.7 rule:** Empirical rule for quick probability estimates
- **Key visual:** The shaded regions illustrate the empirical rule — a z-score of +2.0 is unusual (beyond 95%), while +0.5 is typical.
  - **Teaching tip:** “If you know the mean and SD, you can estimate probabilities without software using this rule.”

## 9 Z-Scores and Probability

**Standard normal distribution** ( $\mu = 0$ ,  $\sigma = 1$ ) allows us to use a single z-table for any normal distribution<sup>[1]</sup>. See Equation 1 for the standard z-score calculation:

$$z = \frac{x - \mu}{\sigma}$$

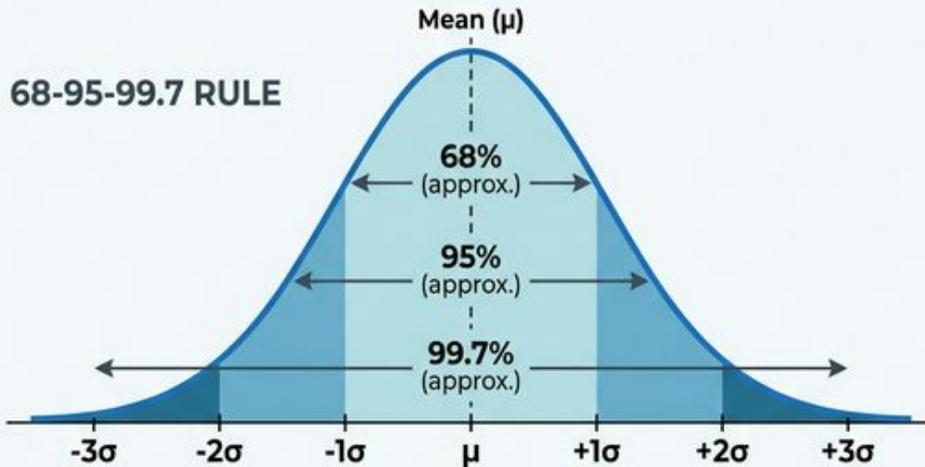
Equation 1: Standard z-score formula

**Example:** Vertical jump heights:  $\mu = 45$  cm,  $\sigma = 7$  cm. What proportion jump higher than 52 cm?

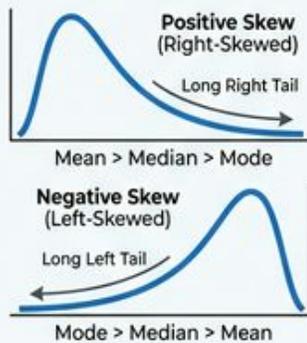
# NORMAL DISTRIBUTION IN KINESIOLOGY

Central Tendency, Symmetry, and Probability Density

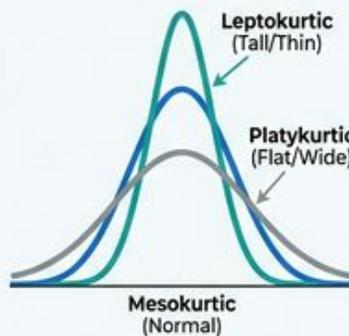
STANDARD NORMAL DISTRIBUTION:  $\mu=0$ ,  $\sigma=1$



## SKEWNESS

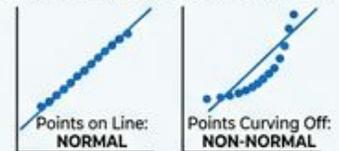


## KURTOSIS

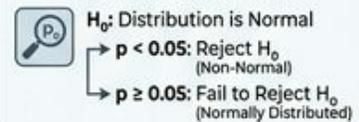


## TESTING NORMALITY

### Q-Q PLOT (QUANTILE-QUANTILE)



### SHAPIRO-WILK TEST



NANO BANANA

Figure 2: Infographic: Key attributes of the Normal Distribution including symmetry, 68-95-99.7 rule, Skewness, Kurtosis, Q-Q plots, and normality testing

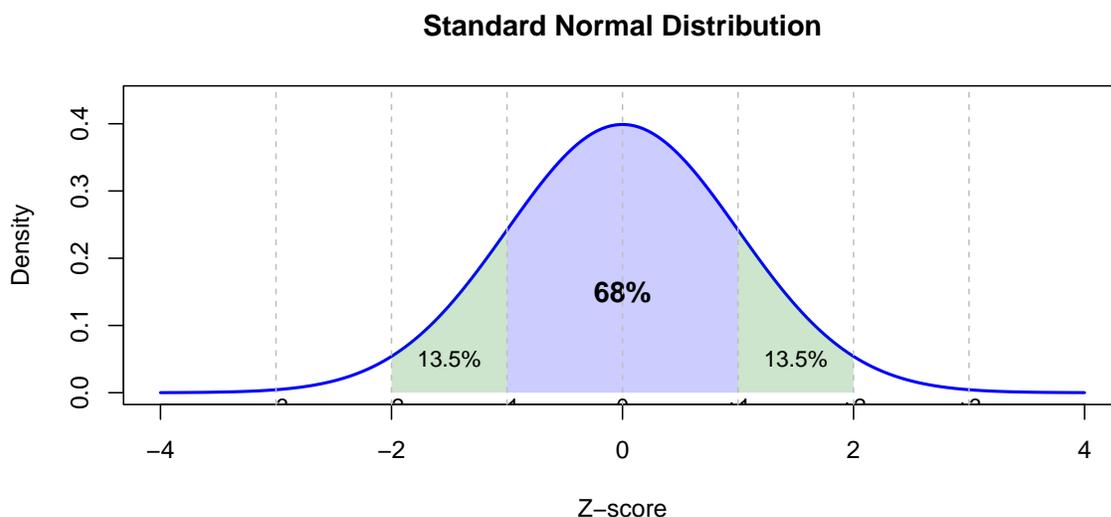


Figure 3: Standard normal distribution showing the 68-95-99.7 rule

**Step 1:**  $z = \frac{52-45}{7} = 1.00$

**Step 2:** Cumulative probability for  $z = 1.00$  is 0.8413 (84.13% below)

**Step 3:** Upper tail:  $P(X > 52) = 1 - 0.8413 = 0.1587$  (**15.87% jump higher**)

How did I get the percentile? I used the [CPT](#).

#### 💡 Real-World Context: Professional Basketball

A study of 53 professional basketball players reported that **Spanish League (LEB)** players had a mean Countermovement Jump (CMJ) of **41.17 cm** during the pre-season (1st assessment).

“The vertical jump is considered a fundamental skill in basketball... [data] allow coaches to compare their players’ performance with high-level athletes.”  
— [Read Abstract](#)

Assuming a hypothetical **SD = 5.0 cm** (consistent with similar cohorts), a **52 cm** jump would have a z-score of:

$$z = \frac{52 - 41.17}{5.0} = 2.17$$

This exceeds **98.5%** of the professional cohort!

- Emphasize: Z-scores standardize different distributions to a common scale.

- Quick check: “If  $z = -1.5$ , is that above or below average?” (Below, 1.5 SDs below mean)

## 10 Skewness: Measuring Asymmetry

**Skewness** quantifies the degree of asymmetry in a distribution<sup>[7,8]</sup>.

**Interpretation:**

- **Skewness = 0:** Symmetric (normal-like)
- **Skewness > 0:** Right-skewed (positive skew); mean > median, long right tail
- **Skewness < 0:** Left-skewed (negative skew); mean < median, long left tail

**Rules of thumb**<sup>[8]</sup>:

Skewness Range	Interpretation
$  \text{Skewness}   < 0.5$	Approximately symmetric
$0.5 \leq   \text{Skewness}   < 1.0$	Moderately skewed
$  \text{Skewness}   \geq 1.0$	Highly skewed

### Movement Science patterns

Reaction times, sway area, and EMG amplitude are **systematically right-skewed** due to physiological lower bounds and occasional large values<sup>[2,6]</sup>.

### Real-World Context

Reaction times in elite sprinters are typically **positively (right) skewed**.

- **Why?** There is a biological “floor” (and IAAF rule) at **0.100 s**—no legitimate reaction can be faster.
- However, there is **no upper ceiling**; a sprinter might react in 0.200 s or slower due to hesitation, creating a long “tail” of slower times to the right.
- **Data:** A study of 1,319 World Championship sprinters found a mean reaction time of **0.166 s** for men, but the distribution leans towards slower outliers. — [Read Abstract](#)

- Teaching tip: Draw a quick sketch on the board showing symmetric, right-skewed, and left-skewed distributions.
- Example: Reaction time cannot be negative but can occasionally be very large (lapses in attention).

## 11 Z-Skew: Statistical Significance

**Z-score for skewness** (z-skew) tests whether observed skewness is statistically different from zero<sup>[7,9]</sup>.

**Formula:**

$$z_{\text{skew}} = \frac{\text{Skewness}}{SE_{\text{skew}}}$$

Equation 2: Z-score for skewness formula

**Decision rules:**

- $|z_{\text{skew}}| < 1.96$ : **Not significant** at  $\alpha = 0.05$  (approximately symmetric)
- $|z_{\text{skew}}| \geq 1.96$ : **Significant** at  $\alpha = 0.05$  (statistically asymmetric)
- $|z_{\text{skew}}| \geq 2.58$ : **Highly significant** at  $\alpha = 0.01$

💡 To make it simple!

If z-skew is in between -2.0 and 2.0, the distribution is approximately symmetric.

**Example 1** (not significant):

- Skewness = 0.45, SE = 0.31  $\rightarrow z_{\text{skew}} = 0.45/0.31 = 1.45$
- Since  $|1.45| < 1.96$ , skewness is **not significant**

**Example 2** (highly significant):

- Skewness = 1.85, SE = 0.31  $\rightarrow z_{\text{skew}} = 1.85/0.31 = 5.97$
- Since  $|5.97| \geq 2.58$ , skewness is **highly significant** ( $p < .01$ )
- Emphasize: Z-skew provides a formal threshold rather than subjective judgment.
- Caveat: With large samples, trivial skewness can become significant; always check magnitude too<sup>[5]</sup>.

## 12 Kurtosis: Tail Weight

**Kurtosis** quantifies the “tailedness” or extremity of a distribution relative to the normal distribution<sup>[10,11]</sup>.

**Types:**

- **Mesokurtic** ( $k = 0$ ): Normal tails
- **Leptokurtic** ( $k > 0$ ): Heavy tails, sharp peak
- **Platykurtic** ( $k < 0$ ): Light tails, flat peak

Rules of thumb:

Kurtosis	Interpretation
$ k  < 1.0$	Normal-like (ok)
$1.0 \leq  k  < 2.0$	Moderate
$ k  \geq 2.0$	Severe

! Important

Modern interpretation emphasizes **tail weight** rather than “peakedness”<sup>[10]</sup>.

- **Heavy tails** (Leptokurtic): Look for bars extending far left/right (outliers are common).
  - **Light tails** (Platykurtic): Look for bars stopping abruptly (outliers are rare/impossible).
- Example: Strength measures under fatigue may show leptokurtosis (heavy tails) due to occasional extreme values from maximal efforts or measurement artifacts<sup>[12]</sup>.
  - **Left plot**: Normal distribution with typical tail behavior
  - **Right plot**: Leptokurtic distribution with more extreme outliers (heavy tails)
  - Teaching tip: “Heavy tails mean more data points far from the mean than expected under normality.”

### 13 Z-Kurtosis: Statistical Significance

**Z-score for kurtosis** (z-kurtosis) tests whether observed kurtosis differs significantly from zero<sup>[7,9]</sup>.

Formula:

$$z_{\text{kurt}} = \frac{\text{Kurtosis}}{SE_{\text{kurt}}}$$

Equation 3: Z-score for kurtosis formula

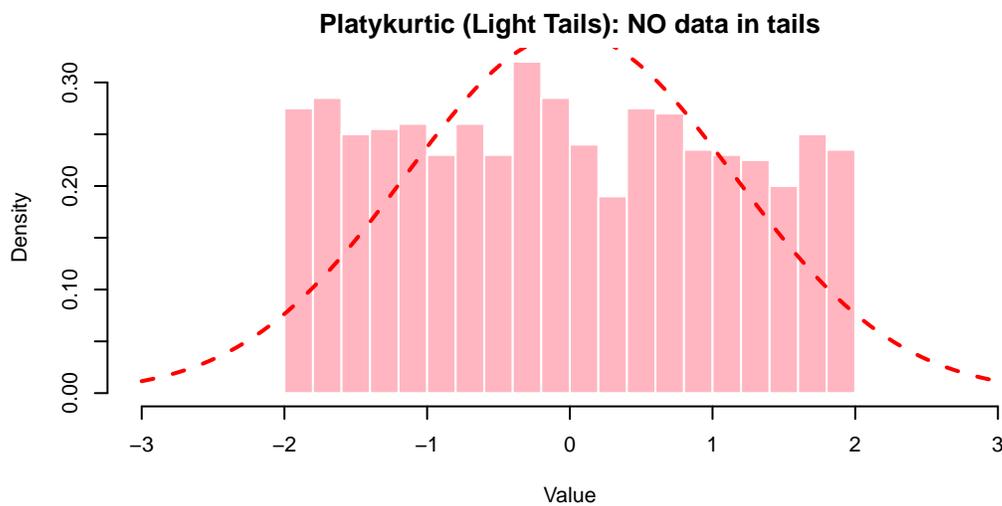
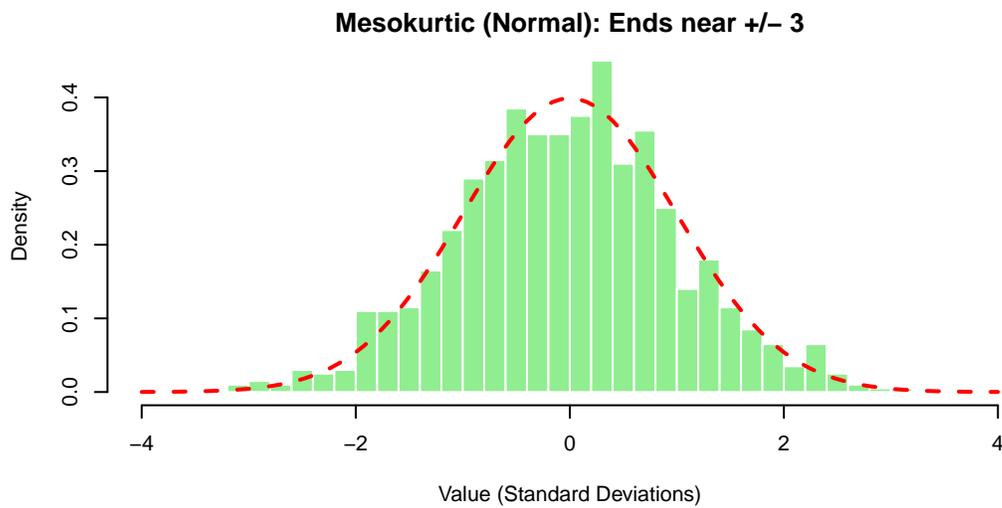
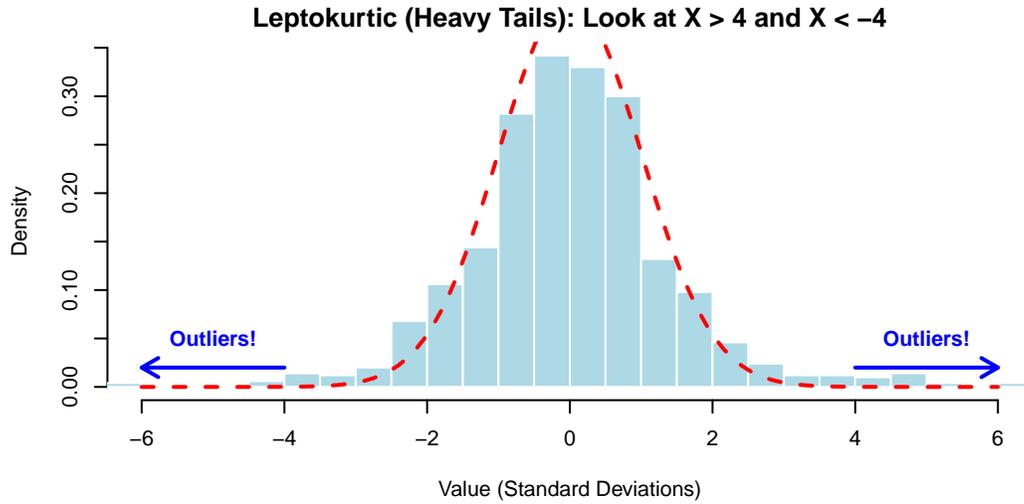


Figure 4: Kurtosis Shapes - LOOK AT THE EDGES!

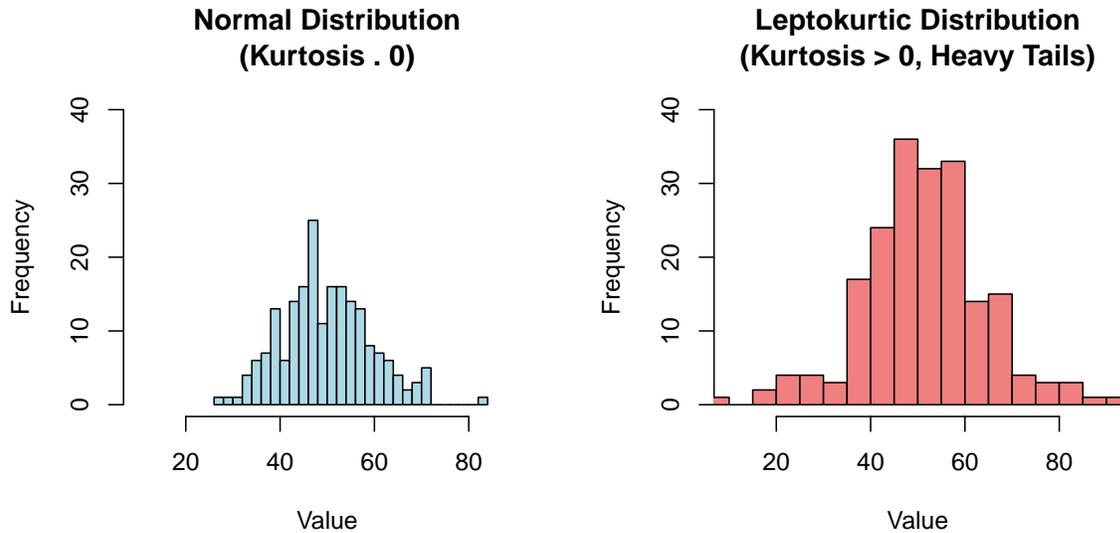


Figure 5: Example of leptokurtic (heavy-tailed) distribution

**Decision rules:**

- $|z_{\text{kurt}}| < 1.96$ : **Not significant** at  $\alpha = 0.05$  (normal-like tails)
- $|z_{\text{kurt}}| \geq 1.96$ : **Significant** at  $\alpha = 0.05$  (significant tail departure)
- $|z_{\text{kurt}}| \geq 2.58$ : **Highly significant** at  $\alpha = 0.01$

💡 To make it simple!  
 If z-kurt is in between -2.0 and 2.0, the distribution is approximately normal.

**Combined interpretation:**

Z-Skew	Z-Kurtosis	Decision
Both $\ z\  < 1.96$	Both $\ z\  < 1.96$	<b>Approximately normal</b>
Either $\ z\  \geq 1.96$	Either $\ z\  \geq 1.96$	<b>Significant departure</b>
Both $\ z\  \geq 2.58$	Both $\ z\  \geq 2.58$	<b>Severe non-normality</b>

- Teaching tip: “Z-scores for shape measures give you objective thresholds, but always pair them with visual assessment.”
- Quick check: “If z-skew = 0.8 and z-kurt = 1.2, what’s your conclusion?” (Both not significant → approximately normal)

## 14 Visual Assessment: Histograms

**Histograms** display the frequency distribution of data by grouping values into bins, revealing the overall shape and patterns that numerical summaries alone cannot capture<sup>[7]</sup>. They allow you to quickly identify:

- **Symmetry or skewness:** Is the distribution balanced or does it lean to one side?
- **Modality:** Are there single or multiple peaks in the data?
- **Outliers:** Are there unusual values far from the main cluster?
- **Spread:** How much variability exists in the data?

Histograms provide an intuitive visual complement to statistics like mean, median, skewness, and kurtosis—helping you see what the numbers are telling you.

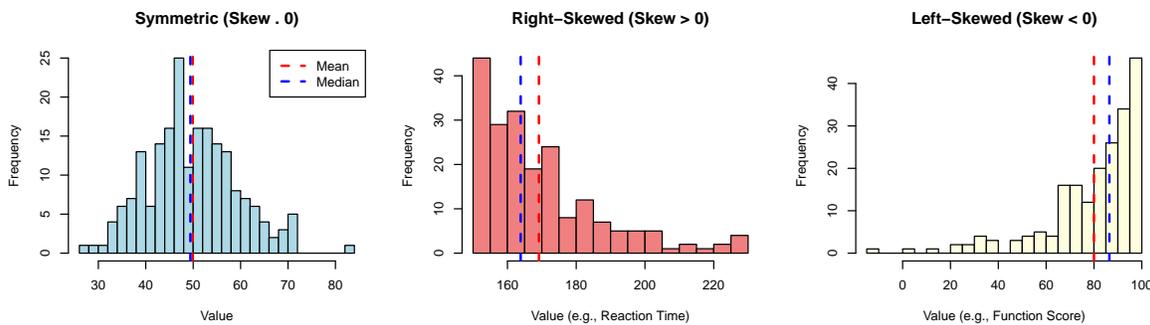


Figure 6: Symmetric, right-skewed, and left-skewed distributions

- Symmetric: Mean and median overlap (red and blue lines nearly identical).
- Right-skewed: Mean  $>$  median; tail extends right (reaction time pattern).
- Left-skewed: Mean  $<$  median; tail extends left (ceiling effect in function scores).

## 15 Visual Assessment: Q-Q Plots

**Q-Q plots** (quantile-quantile plots) compare observed data quantiles to expected normal quantiles<sup>[1,7]</sup>.

**How to interpret:**

- **Points close to diagonal line:** Approximately normal
- **S-curve (lower-left curving up, upper-right curving down):** Right-skewed
- **Inverted S-curve:** Left-skewed
- **Points above line at ends:** Heavy tails (leptokurtic)
- **Points below line at ends:** Light tails (platykurtic)

```

set.seed(123)
par(mfrow = c(1, 3)) # 3 plots side-by-side

# 1. Normal
norm_data <- rnorm(200)
qqnorm(norm_data, main = "Normal", pch = 19, col = "gray50")
qqline(norm_data, col = "red", lwd = 2)

# 2. Right-Skewed (Positive Skew)
# Points curve UP at both ends (convex / U-shape) relative to line
right_skew <- rexp(200, rate = 1)
qqnorm(right_skew, main = "Right-Skewed", pch = 19, col = "gray50")
qqline(right_skew, col = "red", lwd = 2)

# 3. Left-Skewed (Negative Skew)
# Points curve DOWN at both ends (concave / inverted U) relative to line
left_skew <- 100 - rexp(200, rate = 1)
qqnorm(left_skew, main = "Left-Skewed", pch = 19, col = "gray50")
qqline(left_skew, col = "red", lwd = 2)

par(mfrow = c(1, 1)) # Reset layout

```

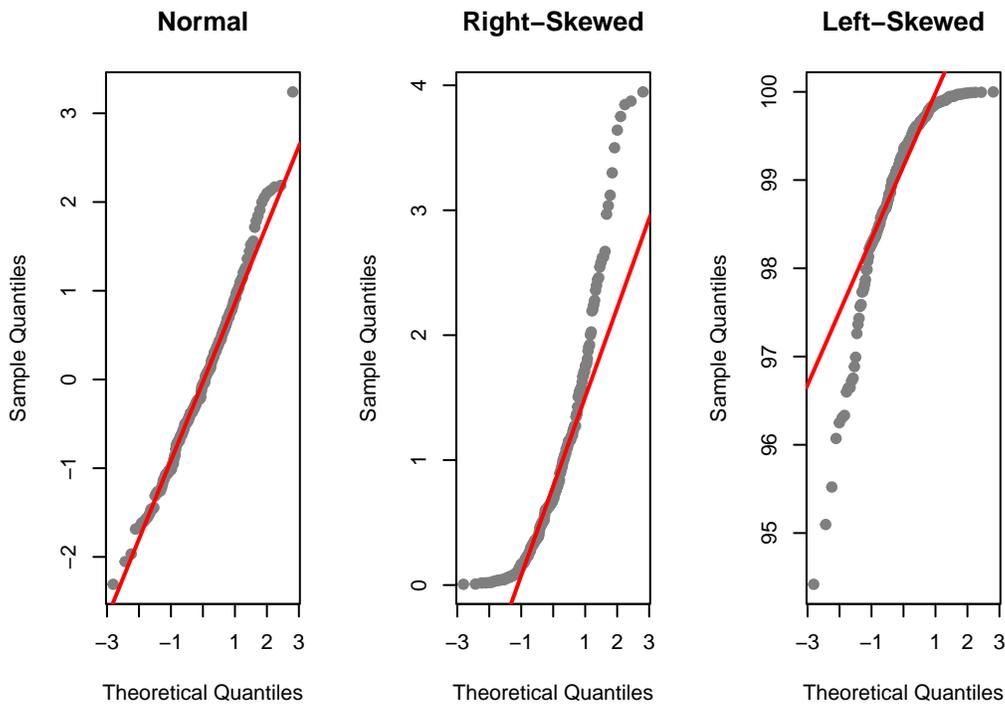


Figure 7: Q-Q Plots: Normal vs. Skewed Distributions

! Gold standard

Q-Q plots are the **most informative visual tool** for assessing normality because they show how data deviate from the normal model across the entire distribution<sup>[7]</sup>.

- Teaching tip: Show students how to create Q-Q plots in SPSS (Analyze → Descriptive Statistics → Explore → Plots → Normality plots with tests).
- Emphasize: “If points follow the line closely with minor random scatter, proceed with parametric methods.”

## 16 Formal Tests: Shapiro-Wilk

**Shapiro-Wilk test** is the most powerful normality test for small to moderate samples<sup>[13,14]</sup>.

**Hypotheses:**

- $H_0$  : Data are normally distributed

- $H$  : Data are not normally distributed

**Decision rule:**

- $p < 0.05$ : Reject  $H$  → **Data are NOT normal** (evidence of non-normality)
- $p \geq 0.05$ : Fail to reject  $H$  → **Data are approximately normal** (no evidence of departure)

 Simple interpretation

- $p < 0.05$  = Data depart significantly from normality
- $p \geq 0.05$  = Data are consistent with normality

**Example:**

- Sprint times:  $W = 0.981$ ,  $p = .448$  → **Data are approximately normal**
- Reaction times:  $W = 0.905$ ,  $p = .001$  → **Data are NOT normal (significant departure)**

 Critical limitation

Do not rely solely on p-values! Sample size strongly affects test results<sup>[5,9]</sup>:

- **Large samples ( $n > 100$ )**: Tests detect trivial departures
- **Small samples ( $n < 30$ )**: Tests lack power to detect real departures

So, always combine formal tests with visual assessment (Q-Q plots, histograms) to make informed decisions about normality.

## 17 The Sample Size Paradox

Why visual and formal methods often conflict:

### 17.1 Large Samples ( $n > 100$ )

- Formal tests become **hypersensitive**
- Detect trivial departures (statistically significant but practically irrelevant)
- Q-Q plot shows near-perfect normality, but  $p < 0.05$

**Decision:** Trust visual assessment

## 17.2 Small Samples ( $n < 30$ )

- Formal tests have **low power**
- Fail to detect real departures (statistically non-significant but practically important)
- Q-Q plot shows clear departure, but  $p > 0.05$

**Decision:** Trust visual assessment

### ! Important

Modern statistical practice **prioritizes visual assessment** with formal tests serving as supplementary evidence<sup>[5,7,12]</sup>.

**Key message:** Due to sample size effects, formal tests work best with moderate samples ( $n = 30-100$ ) but are problematic with very small or very large samples. **Always start with visual assessment (Q-Q plots, histograms) regardless of sample size.**

:::

## 18 Decision Framework: Integrating Evidence

When visual and formal methods conflict, follow this hierarchical approach<sup>[5,7]</sup>:

**Step 1: Prioritize visual assessment** (reveals nature and magnitude)

**Step 2: Consider sample size** when interpreting formal tests

- $n < 30$ : Weight visual heavily (tests underpowered)
- $30 < n < 100$ : Balance visual and formal (tests most informative)
- $n > 100$ : Weight visual heavily (tests hypersensitive)

**Step 3: Evaluate practical vs. statistical significance**

- Are z-skew and z-kurt in between  $\pm 2$  (acceptable) or beyond  $\pm 2$  (significant)?
- Do Q-Q plots show systematic departure or minor waviness?

**Step 4: Apply convergence rule**

- **All agree** (visual, z-scores, formal test)  $\rightarrow$  Proceed with high confidence
- **Visual shows normality BUT  $p < .05$**   $\rightarrow$  Likely trivial departure (large  $n$ ) — use parametric
- **Visual shows departure BUT  $p > .05$**   $\rightarrow$  Likely real problem (small  $n$ ) — use transformation/nonparametric
- **Mixed/borderline evidence**  $\rightarrow$  Use robust methods (Welch's t-test, bootstrap)

- Emphasize: “Integration, not exclusion — use all available evidence.”
- Teaching prompt: Present a scenario and have students apply the framework.

## 19 Convergence Rule: Practical Examples

Apply the convergence rule to resolve conflicts between methods:

### Scenario 1: Large Sample (n = 150)

- Q-Q plot: Points hug the line closely
- z-skew = 0.8, z-kurt = 1.1 (both in  $\pm 2$ )
- Shapiro-Wilk:  $W = 0.975$ ,  $p = .018$

#### Decision: Use parametric

- Visual shows near-perfect normality
- z-scores show acceptable shape
- Significant p-value likely due to large n detecting trivial departure

### Scenario 2: Small Sample (n = 22)

- Q-Q plot: Clear S-curve (right-skewed)
- z-skew = 2.8 (beyond  $\pm 2$ )
- Shapiro-Wilk:  $W = 0.918$ ,  $p = .062$

#### Decision: Transform or use nonparametric

- Visual clearly shows departure
- z-skew confirms significant skewness
- Non-significant p-value due to low power (small n)
- Don't let  $p > .05$  mislead you!

#### ! Key insight

When methods conflict, **trust the pattern across multiple indicators** rather than relying on a single test. Large samples reveal trivial issues; small samples hide real problems.

- Walk through both scenarios step-by-step
- Emphasize: “This is why we integrate evidence—no single test tells the whole story”
- Real-world tip: “Document your reasoning in your analysis notes”

## 20 Decision Table for Conflicts

Visual Assessment	Formal Test	Sample Size	Recommended Action
Approximately normal	$p < 0.05$	$n < 30$	Use parametric (test underpowered)
Approximately normal	$p < 0.05$	$n \geq 100$	Use parametric (trivial departure)
Clear departure	$p > 0.05$	$n < 30$	Transform/nonparametric (test underpowered)
Clear departure	$p > 0.05$	$n \geq 100$	Investigate data quality
Mild departure	$p < 0.05$	Any	Use robust methods (Welch's t)
Severe departure	$p < 0.05$	Any	Transform/nonparametric

### Practical checklist

When reviewing SPSS output, systematically check:

Sample size, Visual (Q-Q + histogram), Magnitude (z-skew, z-kurt), Formal test (Shapiro-Wilk), **Integrated decision**

- Teaching tip: “Use this table as a reference when analyzing your own data — don’t memorize, apply thoughtfully.”

## 21 Common Non-Normal Patterns

Movement Science data often show systematic departures from normality<sup>[3,4]</sup>:

1. **Right-skewed:** Reaction time, sway area, EMG amplitude (physiological lower bounds)<sup>[6]</sup>
2. **Ceiling/floor effects:** Function scores, pain scales (clustering at boundaries)
3. **Bimodal:** Mixed groups (trained vs. untrained) — analyze separately<sup>[15]</sup>
4. **Heavy-tailed:** Outlier-prone measures (strength under fatigue, motivation artifacts)<sup>[12]</sup>

### **i** Real example

A researcher collects reaction times: strong right skew (skew = 1.8), Shapiro-Wilk  $p = .002$ , Q-Q plot shows clear upward curvature.

#### **Options:**

1. Log transform  $\rightarrow$  confirm normality on log scale  $\rightarrow$  proceed with parametric methods
2. Report median/IQR instead of mean/SD
3. Use nonparametric tests (Mann-Whitney U, Kruskal-Wallis)

- Emphasize: “Recognizing these patterns helps determine whether transformations, robust methods, or nonparametric approaches are preferable.”

## 22 When Normality Matters Most

**High priority:** Check normality carefully

- Small samples ( $n < 30$ ): Parametric methods rely heavily on distributional assumptions<sup>[1]</sup>
- Hypothesis tests with p-values near cutoffs ( $p \approx 0.05$ ): Violations could tip decisions<sup>[9]</sup>
- Variables known to be non-normal: Reaction time, sway area, EMG amplitude<sup>[6]</sup>

**Lower priority:** Normality less critical

- Large samples ( $n > 100$ ): Central Limit Theorem makes parametric tests robust<sup>[5]</sup>
- Robust methods: Welch’s t-test, bootstrapping, rank-based tests tolerate departures<sup>[12,16]</sup>
- Descriptive summaries: Report median/IQR regardless of normality<sup>[7]</sup>

### **!** Key principle: context over rules

There is no universal “how normal is normal enough.” The answer depends on sample size, purpose (inference vs. description), magnitude of departure, and robustness of method<sup>[5,16]</sup>.

- Teaching tip: “Think critically about your analysis context rather than applying blanket rules.”

## 23 What to Do When Data Are Not Normal

When departures are consequential, consider these principled options<sup>[4,12]</sup>:

1. **Use robust or nonparametric methods:**

- Median/IQR instead of mean/SD
- Mann-Whitney U or Kruskal-Wallis instead of t-tests/ANOVA
- Bootstrap or permutation tests

2. **Transform the variable:**

- Log transformation for right-skewed data (reaction time, sway area)<sup>[6]</sup>
- Square root or Box-Cox for count data

3. **Accept departure and proceed with caution:**

- Many procedures are robust to moderate departures ( $n > 30$ , balanced designs)<sup>[5]</sup>
- Use Welch's t-test when variances differ<sup>[16]</sup>

4. **Separate subgroups:** If bimodal, analyze groups separately<sup>[15]</sup>

5. **Acknowledge and report:** Describe distributional shape and justify your approach<sup>[7]</sup>

- Emphasize: “Your decision should be data- and context-driven, not reflexive.”

## 24 Worked Example: Complete Assessment

**Scenario:** Sprint times from 40 participants

**Step 1:** Visualize (histogram + Q-Q plot)

- Histogram: Reasonably symmetric, no extreme outliers
- Q-Q plot: Points close to diagonal with minor waviness

**Step 2:** Compute shape measures

- Skewness = 0.15 (negligible)
- Kurtosis =  $-0.22$  (close to normal)
- z-skew =  $0.15 / 0.26 = 0.58$  (not significant)
- z-kurt =  $-0.22 / 0.51 = -0.43$  (not significant)

**Step 3:** Run formal test

- Shapiro-Wilk:  $W = 0.981$ ,  $p = .448$  (do not reject normality)

**Step 4:** Integrated decision

- All evidence converges: **approximately normal**
- **Conclusion:** Proceed with parametric methods (t-tests, ANOVA) confidently

- Teaching tip: Walk through this example step-by-step, showing SPSS output at each stage.
- Emphasize: “This is the workflow you should follow for every normality assessment.”

## 25 Summary: Key Takeaways

1. **Normal distribution:** Symmetric, bell-shaped; defined by  $\mu$  and  $\sigma$ ; foundational for inference
2. **68-95-99.7 rule:** Quick probability estimates without software
3. **Skewness & kurtosis:** Quantify shape; use z-scores for significance testing
4. **Visual assessment is primary:** Q-Q plots and histograms reveal nature and magnitude of departures
5. **Formal tests are supplementary:** Sample size strongly affects results (hypersensitive with large  $n$ , underpowered with small  $n$ )
6. **Integration over exclusion:** When conflicts arise, combine all evidence using the decision framework
7. **Context matters:** Sample size, purpose, magnitude, and robustness of methods determine whether departures are consequential
8. **Multiple options exist:** Transformation, robust methods, nonparametric tests when normality fails

### ! Important

The goal is not to worship normality or avoid it reflexively, but to treat it as **one useful model among many**, applicable when data and context support it<sup>[12,15]</sup>.

## 26 Practice Questions

1. What is the most distinctive property of the normal distribution?
2. If  $z = 1.5$ , approximately what percentile is this?
3. What does positive skewness indicate? Give a Movement Science example.
4. When do visual and formal normality assessments typically conflict, and why?
5. What is the decision rule for z-skew at  $\alpha = 0.05$ ?
6. If Q-Q plot shows approximate normality but Shapiro-Wilk  $p = .02$  with  $n = 150$ , what should you do?
7. Name three common non-normal patterns in Movement Science data.
8. When is normality assessment most critical (high priority)?

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